Bose polarons at finite temperature and strong coupling

Pietro Massignan

UPC & ICFO - Barcelona

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

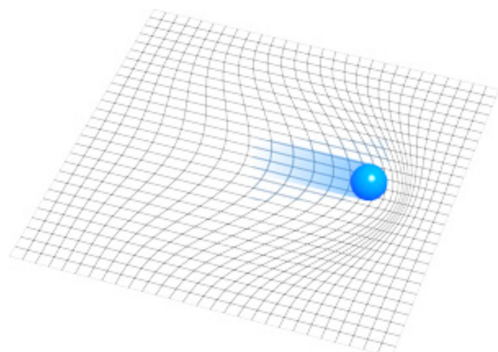
UPC

ICFO

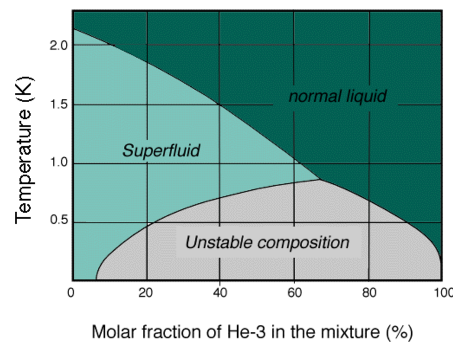
Institut de Ciències Fotòniques

The impurity problem

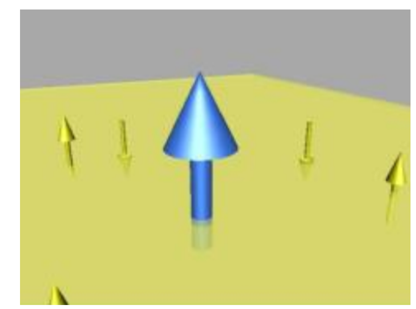
✦ Dilute impurities in quantum many-body systems



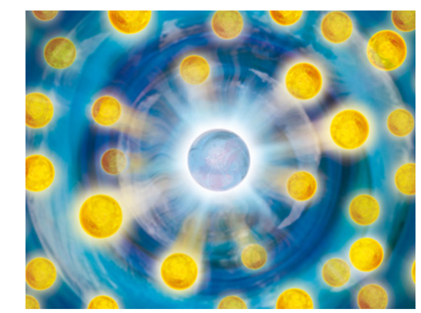
electrons
in a phonon bath



^{3}He in ^{4}He



magnetic impurities
(Kondo problem)



ultra-dilute
quantum mixtures

✦ probes of equilibrium and non-equilibrium properties: quasi-particle energies, lifetimes, effective masses, coherence, formation times, orthogonality catastrophe, …
MIT [PRL '09], ENS [PRL '09], Innsbruck [Nature 2012, Science 2017], Cambridge [Nature 2012], LENS [PRL 2018], Aarhus [PRL 2016], JILA [PRL 2016], …

✦ ideal tools to manipulate and study a many-body system (transport, topology, …)

✦ well-posed problem (QMC, variational, diagrammatic, RG, functional determinants, …)

Recent reviews: PM, Zaccanti and Bruun [Rep. Prog. Phys 2014], Schmidt, Knap *et al.* [Rep. Prog. Phys. 2018]

Outline

- Impurities in a weakly-interacting Bose gas ("Bose polarons")
  - Part I:
    a new quasi-particle appears at non-zero temperature
  - Part II:
    an *Anderson orthogonality catastrophe* arises in ideal BECs

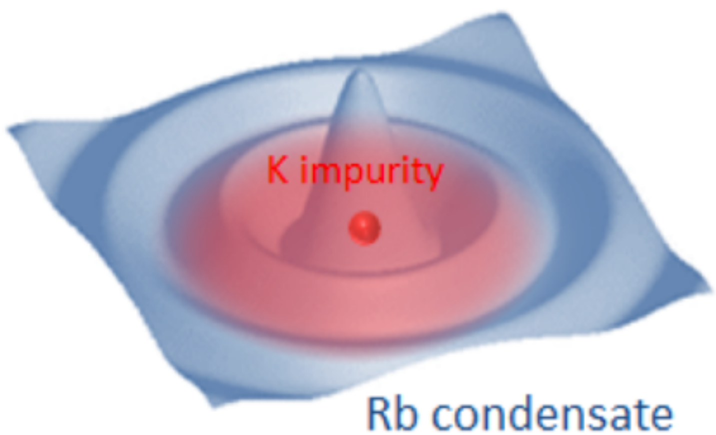
Nils Guenther

Maciej Lewenstein

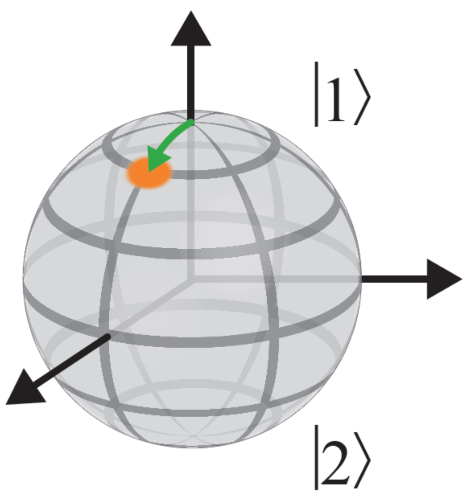
Georg M. Bruun

Richard Schmidt

Impurities in a Bose gas



JILA: Hu, …, Cornelll and Jin [PRL 2016]

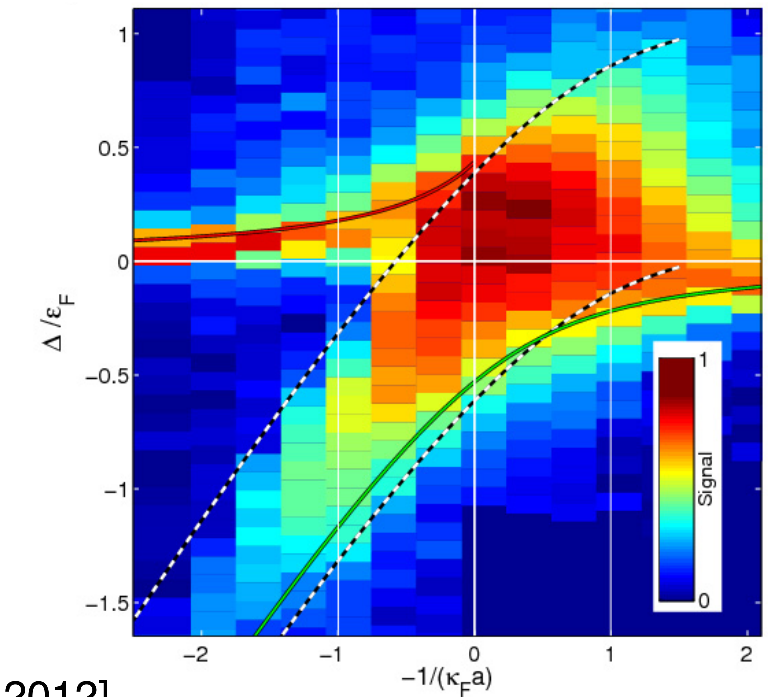


weak RF pulse + quick decoherence:
a few $|2\rangle$ impurities in a bath of $|1\rangle$ atoms

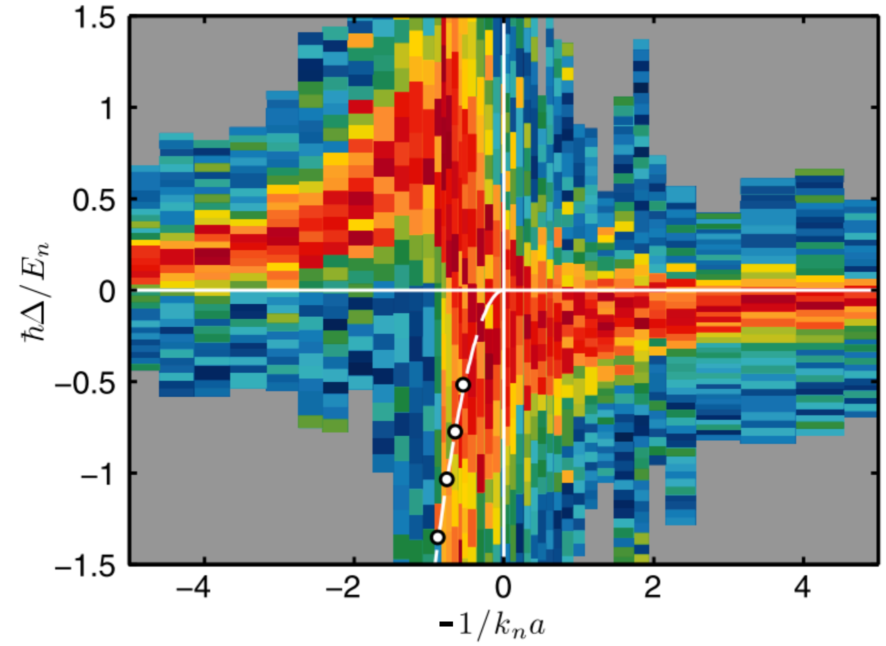
Aarhus: Jørgensen, …, Bruun and Arlt [PRL 2016]

T=0 theory: Rath, Schmidt, Das Sarma, Bruun, Levinsen, Parish, Demler, Peña-Ardila, Giorgini, Pohl, Camacho-Guardian, …

Polarons:
Fermi vs. Bose



[Innsbruck 2012]



[Aarhus 2016]

Differences	non-interacting Fermi sea	weakly-interacting Bose gas ($k_n a_B \ll 1$)
Temperature	smooth crossover from degenerate to classical	BEC phase transition at T_c
Impurity ground state	polaron/molecule transition	smooth crossover
Three-body physics	negligible	important role
Stability	(meta-)stable mixture	rapid three-body losses

Part I

Finite-T analysis

Definition of the problem

- Weakly-interacting BEC, treated with Bogoliubov theory

» condensate density: $n_0 = n[1 - (T/T_c)^{3/2}]$

» critical temperature: $T_c = \frac{2\pi}{m_B}\left(\frac{n}{\zeta(\frac{3}{2})}\right)^{2/3} \approx 0.436 E_n$

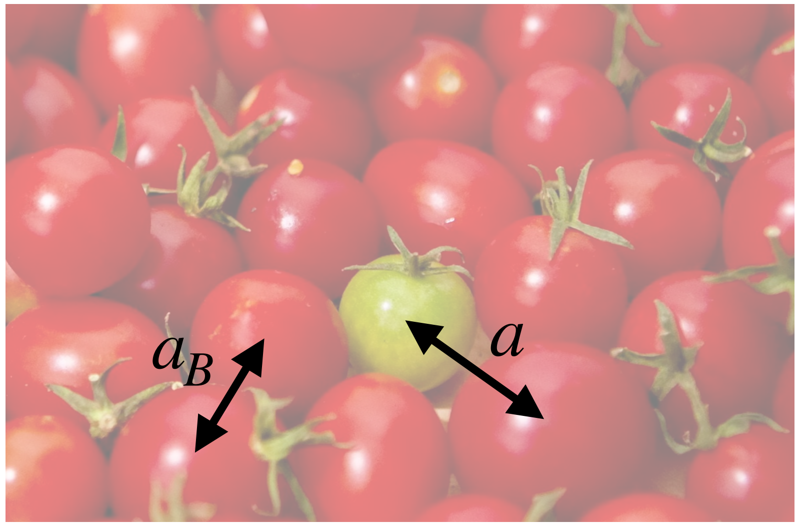
» boson-boson vacuum scattering: $\mathcal{T}_B = 4\pi a_B/m_B$

» BEC chemical potential: $\mu_B = \mathcal{T}_B n_0$

» Bogoliubov excitations: $E_{\mathbf{k}} = \sqrt{\epsilon^B_{\mathbf{k}}(\epsilon^B_{\mathbf{k}} + 2\mu_B)}$

» free bosons: $\epsilon^B_{\mathbf{k}} = k^2/2m_B$

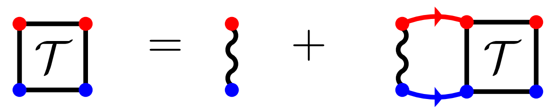
units: $k_n = (6\pi^2 n)^{1/3}$
$E_n = k_n^2/2m_B$



- Impurity-bath coupling: s-wave contact interaction with scattering length a
- **Finite temperature Green's functions:**
    - Polaron energy: $\omega_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \mathrm{Re}[\Sigma(\mathbf{p}, \omega_{\mathbf{p}})]$
    - Polaron residue: $Z_{\mathbf{p}} = \frac{1}{1 - \partial_\omega \mathrm{Re}[\Sigma(\mathbf{p}, \omega)]|_{\omega_{\mathbf{p}}}}$

Diagrammatic scheme

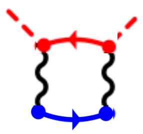
$$\text{Impurity Green's function: } \mathcal{G}(\mathbf{p}, i\omega_j) = \frac{1}{\mathcal{G}_0(\mathbf{p}, i\omega_j)^{-1} - \Sigma(\mathbf{p}, i\omega_j)}$$



$$\text{Ladder T-matrix: } \mathcal{T}(\mathbf{p}, i\omega_j)^{-1} = \mathcal{T}_v^{-1} - \Pi(\mathbf{p}, i\omega_j)$$

at T>0, important diagram

missing in ladder approx:



T=0 ladder: Rath and Schmidt, PRA 2013

Perturbation theory at T>0: Levinsen, Parish, Christensen, Arlt, and Bruun, PRA 2017

Extended T>0 diagrammatic scheme: Guenther, PM, Lewenstein and Bruun, PRL 2018

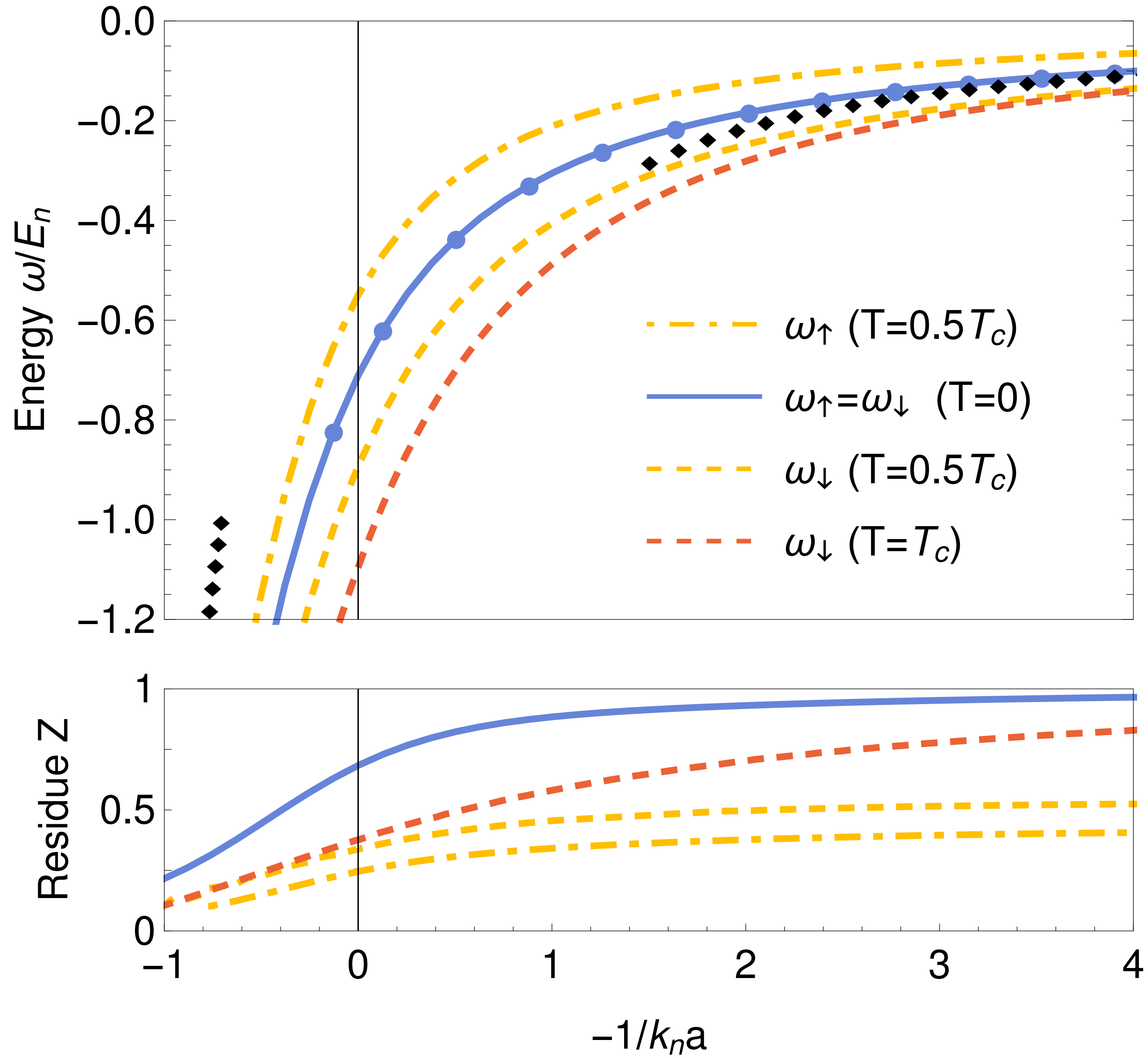
Varying coupling strength

Aarhus: $k_n a_B = 0.01$

$T\!=\!0$

$T\!=\!0.5T_c$

$T\!=\!T_c$



black diamonds: mean-field

blue dots: T=0 ladder theory

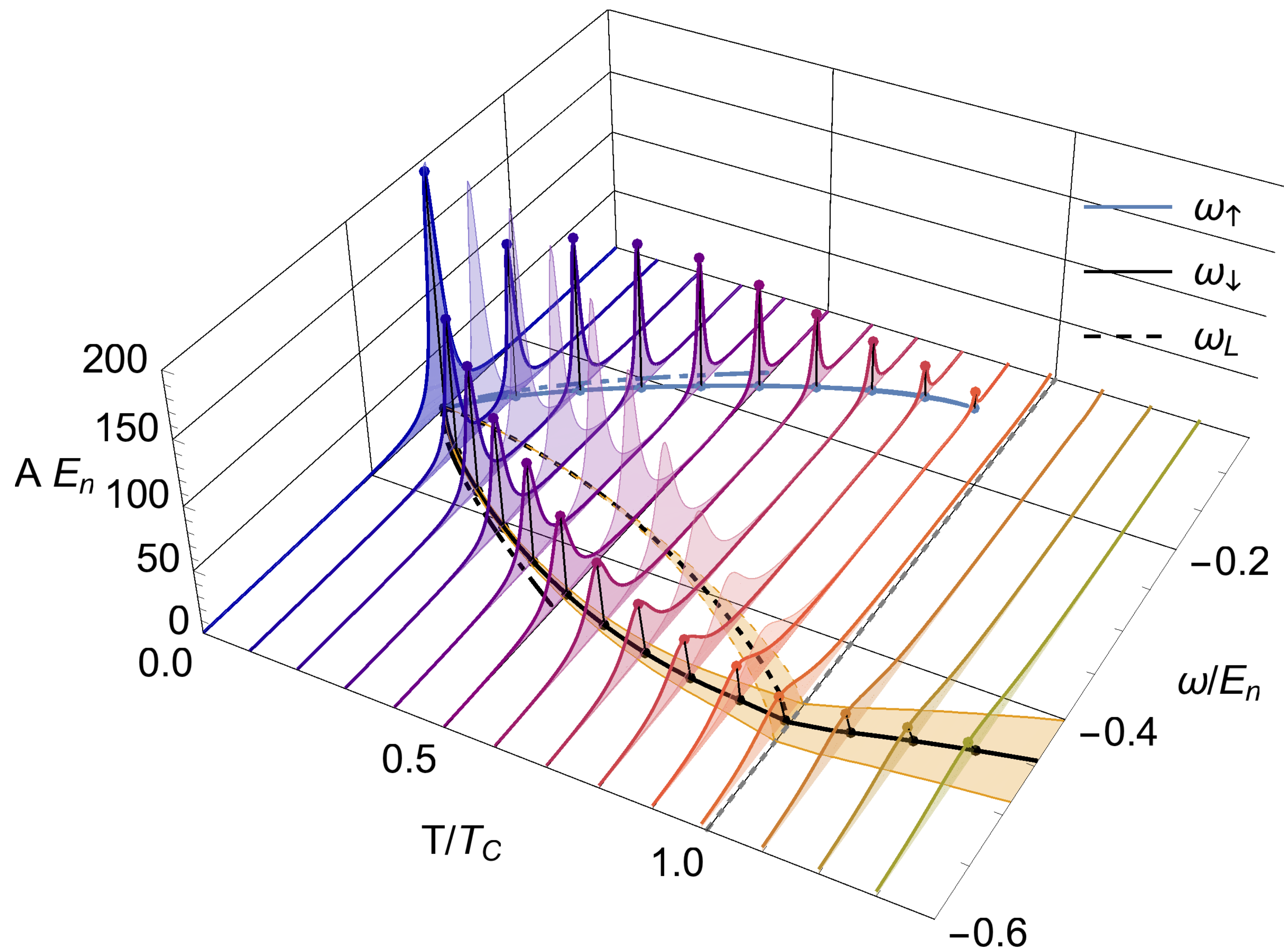
[Rath and Schmidt, PRA 2013]

Guenther, PM, Lewenstein, and Bruun
Phys. Rev. Lett. **120**, 050405 (2018)

Increasing temperature

Weak attraction
$(k_n a = -1)$

Spectral function:



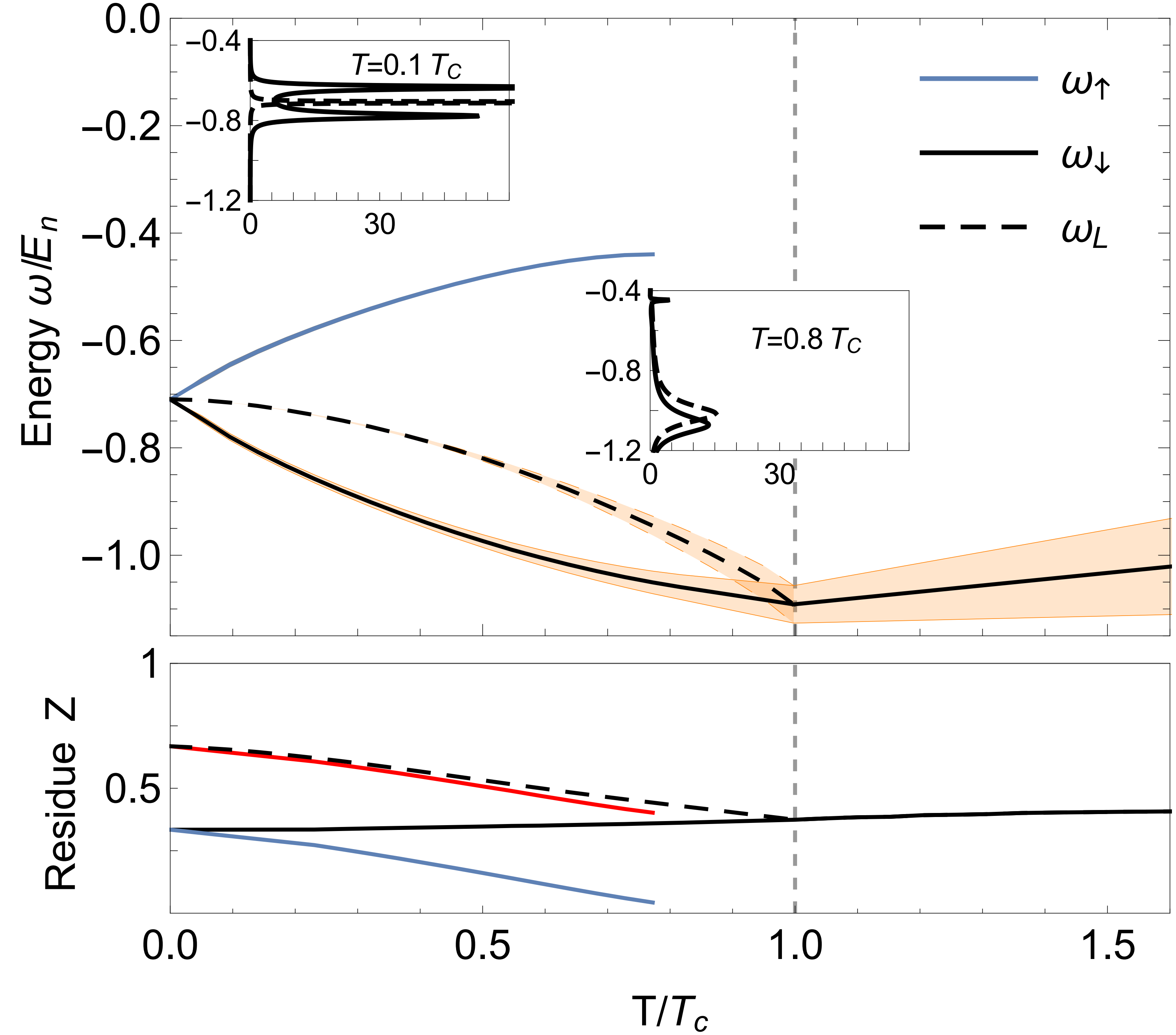
Guenther, PM, Lewenstein, and Bruun
Phys. Rev. Lett. **120**, 050405 (2018)

Increasing temperature

Strong attraction
(unitarity)

Energy:

Residue:

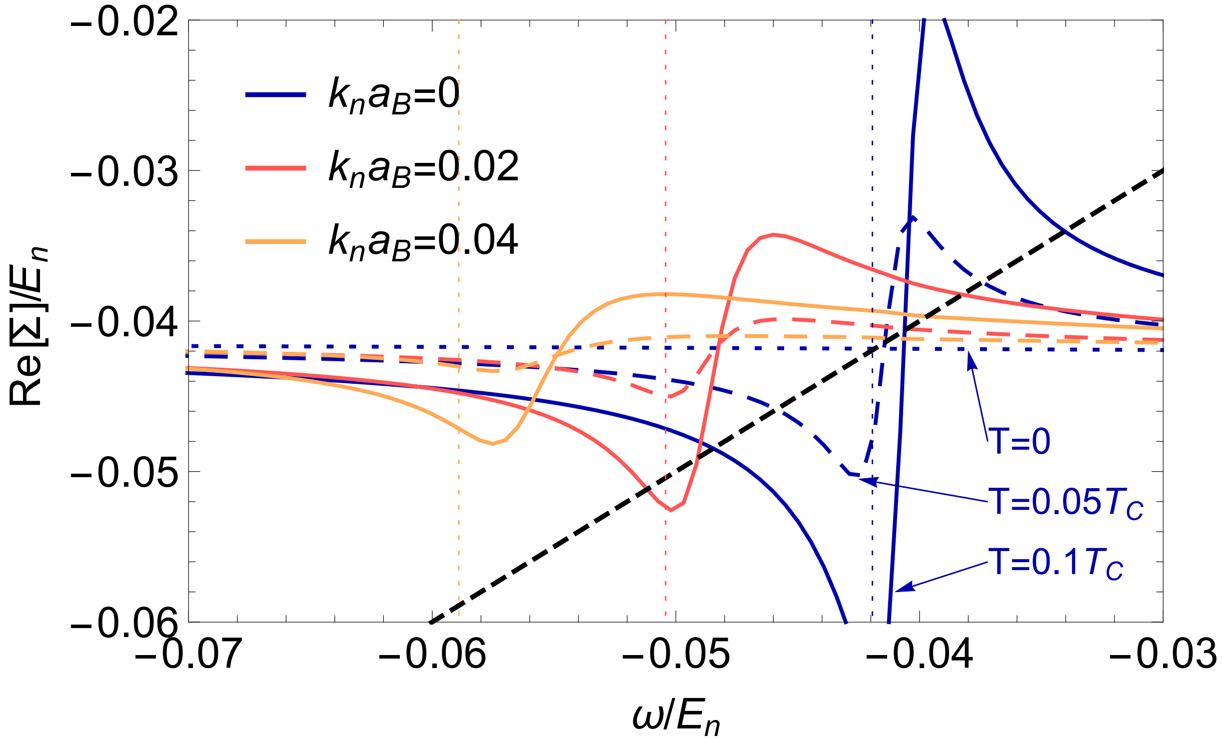


Understanding fragmentation

Polaron energy: $\omega_0 = \mathrm{Re}[\Sigma(\mathbf{p}=0,\omega_0)]$

Re(Σ) at weak coupling & low-T:

$k_n a = -0.1$

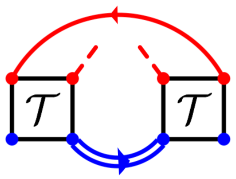


$$\Sigma_1(\omega) \approx \int \frac{d^3k}{(2\pi)^3} \frac{f_{\mathbf{k}}}{\mathcal{T}_v^{-1} - \Pi(\mathbf{k},\omega+E_{\mathbf{k}}) - \frac{n_0}{\omega+E_{\mathbf{k}}-\epsilon_{\mathbf{k}}}}$$

on-shell for: $\omega + E_{\mathbf{k}} = \epsilon_{\mathbf{k}} + \Sigma_0(\mathbf{k},\omega+E_{\mathbf{k}})$

$$\approx \frac{\omega + n_0\mathcal{T}_B}{\omega - n_0(\mathcal{T}_v - \mathcal{T}_B)} n_{\mathrm{ex}}\mathcal{T}_v$$

non-condensed fraction



$|a| \gtrsim a_B$: equal splitting

$$\omega_{\uparrow,\downarrow} \simeq \omega_0[1 \pm (Z_0 n_{\mathrm{ex}}/n_0)^{1/2}]$$
$$Z_{\uparrow,\downarrow} \simeq Z_L/2$$

$|a| \lesssim a_B$: single polaron

$$\omega_\uparrow \simeq n\mathcal{T}_v$$

in accord with perturbation theory [Levinsen et al., PRA 2017]

General features

- Strong temperature dependence, due to the Bogolubov spectrum crossing over from linear to quadratic
  (at $T=T_c$, and at $\epsilon_{\mathbf{k}}^B \approx 2\mu_B$)

- Similar scenario whenever the bath undergoes a phase transition breaking a continuous symmetry

- Examples: normal and high-T_c superconductors, ^{3}He-^{4}He mixtures, ultracold fermionic superfluids, magnetic systems, …

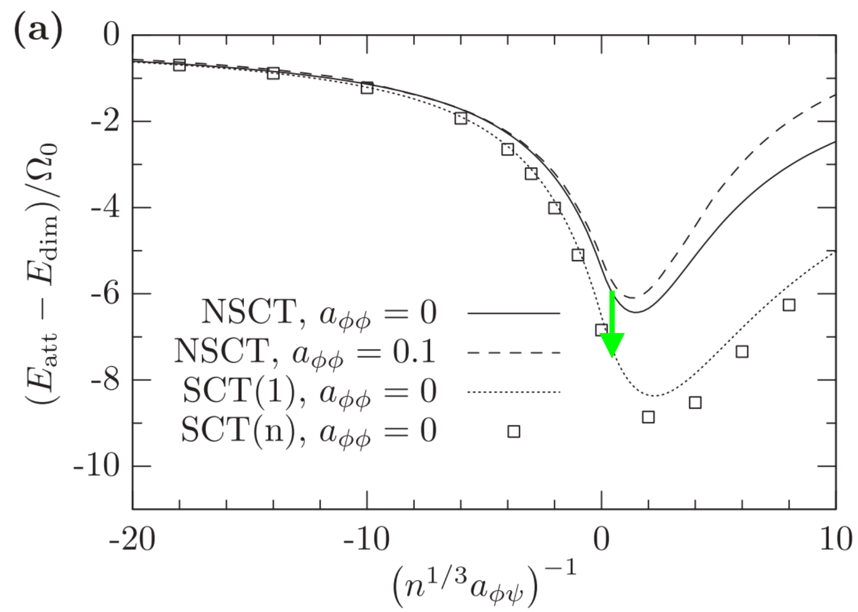
- The new quasiparticle emerges due to the coupling between the impurity and a large number of low-energy soft excitations
  (like Landau damping in ordinary plasmas,
  and plasminos in Yukawa and QED theories)

Baym, Blaizot, and Svetitksy, PRD 1992
Braaten, Astrophys. J. 1992

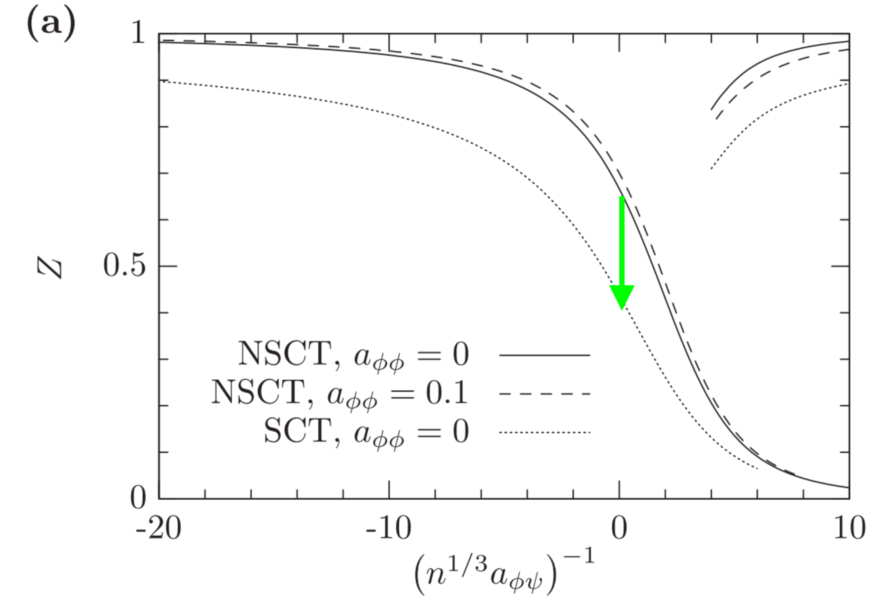
Validity of the 1PH-approx for an ideal BEC?

equal masses at unitarity

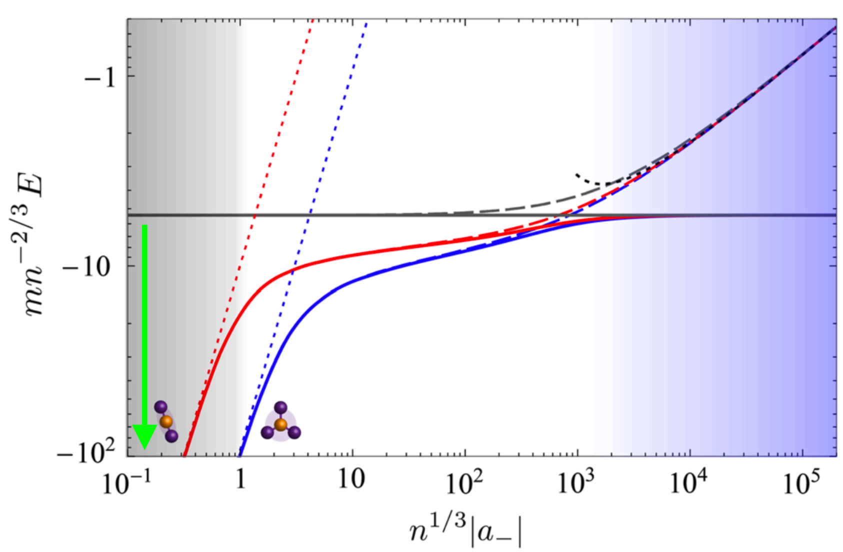
Energy

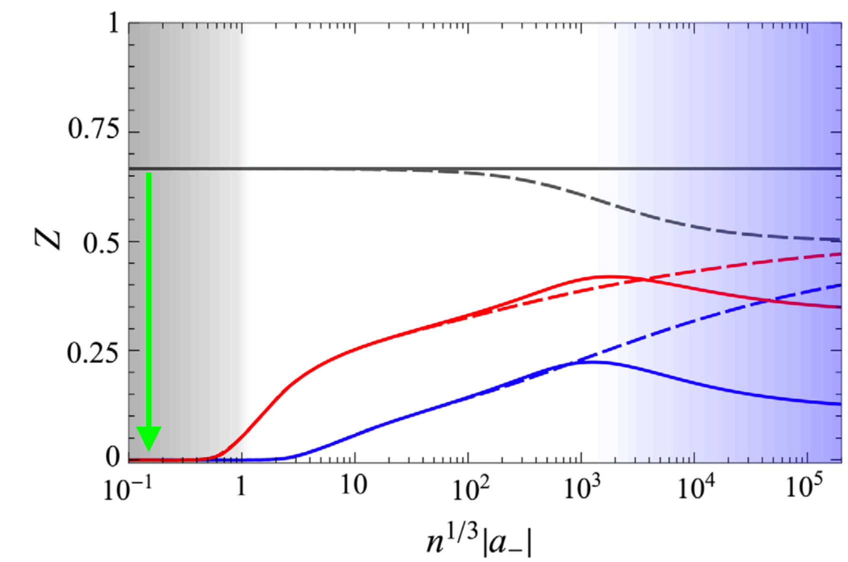


Residue



Rath and Schmidt, PRA 2013





Yoshida, Endo, Levinsen and Parish, PRX 2018

Part II

Infinitely-massive impurities

Ideal BEC

- Imp+boson inside a sphere of radius R; non-interacting: $\psi(r) = \frac{1}{\sqrt{2\pi R}} \frac{\sin(k_0 r)}{r}$

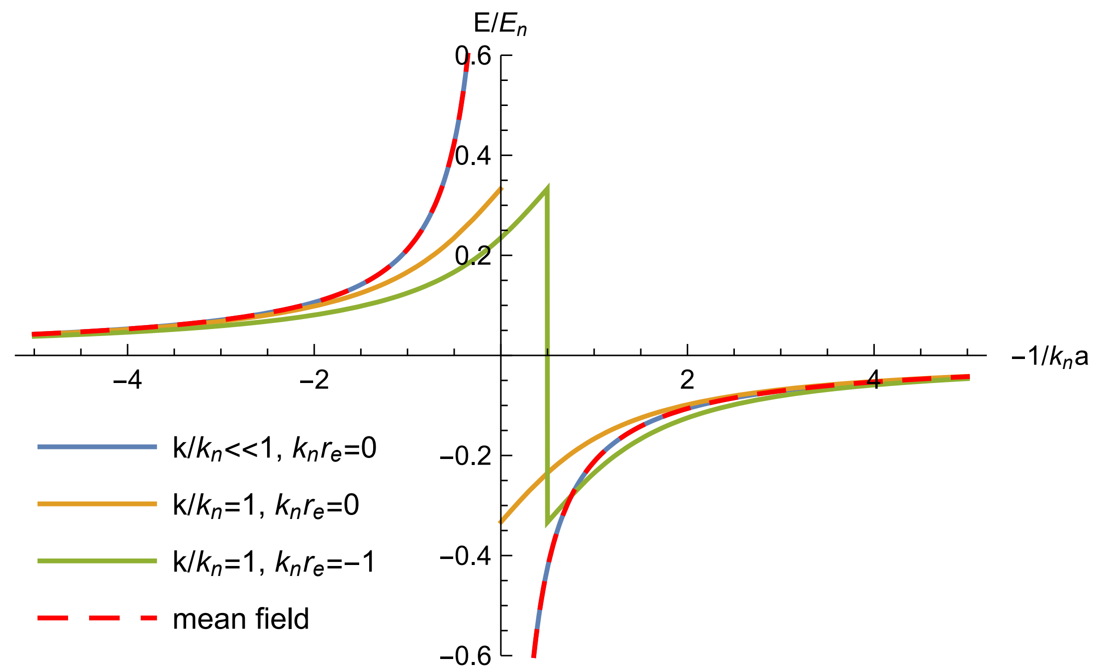
- Adding a short-ranged interaction: $\psi(r) = \frac{1}{\sqrt{2\pi R}} \frac{\sin(kr + \delta)}{r}$

- Energy shift: $\Delta E = \frac{\hbar^2}{m} k \, \Delta k = \frac{2\pi\hbar^2}{m_r} \left(-\frac{\delta}{k} \right) |\psi(0)|^2$

- Phase shift: $\cot\delta = -\frac{1}{ka} + \frac{r_e k}{2} + O(k^3)$

- Weak interaction: $\delta = -ka$ (MF shift)

- Unitarity: $\delta = \pi/2$
  taking $k = 1/\xi$ one has $\Delta E = -\frac{1}{3}(k_n \xi) E_n$
  which diverges for an ideal gas!



Ideal BEC

- The BEC is a product state: $|\Psi\rangle = \prod_N |\psi\rangle$
- If $|\psi\rangle \neq |\psi_0\rangle$, then $z = |\langle\psi_0|\psi\rangle|^2 < 1$
- And in the thermodynamic limit the residue $Z = z^N \to 0$
- For *every* bath-impurity interaction!
- Ideal BEC + ∞-mass impurity ➜ Anderson orthogonality catastrophe

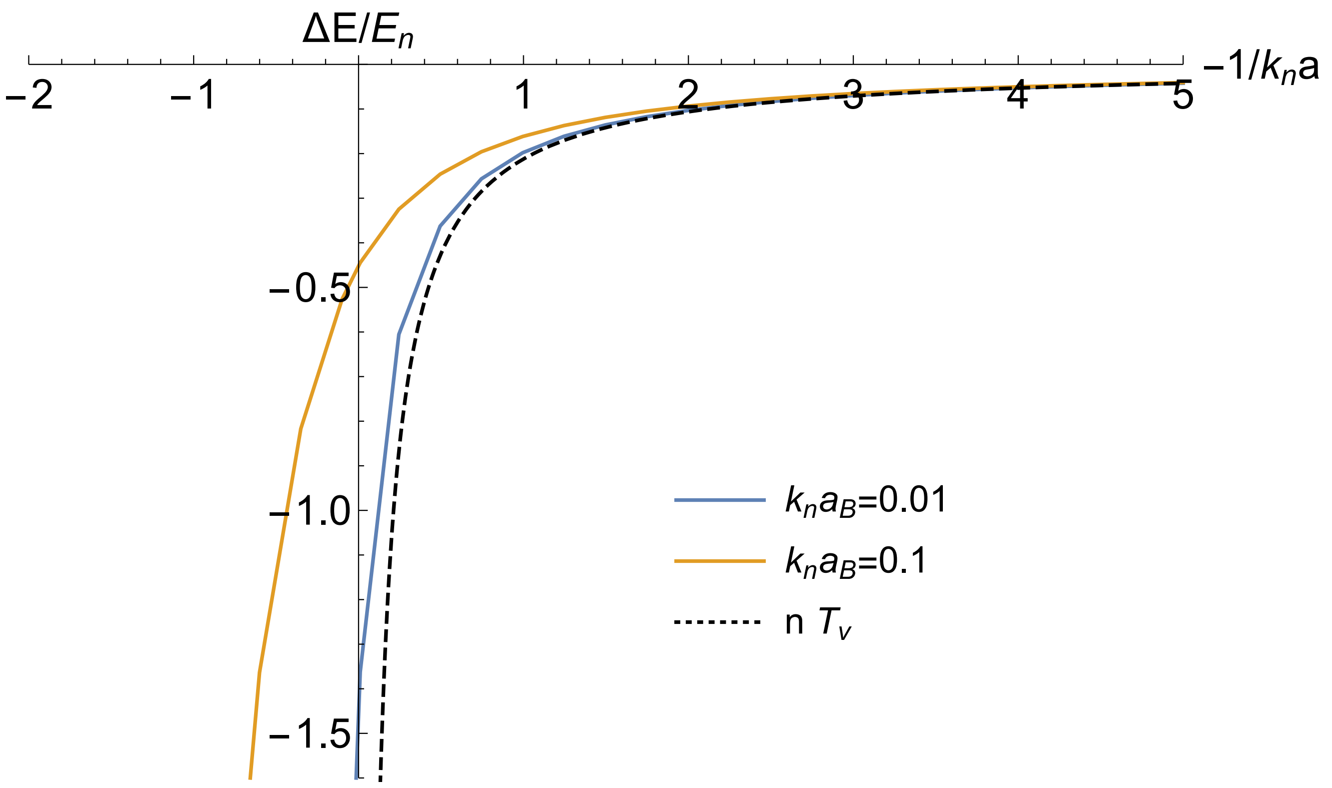
Weakly-interacting BEC

- GPE equation for the radial wavefunction $\psi(r) = \dfrac{\chi(r)}{r}$

- BEC-impurity potential: attractive square well

- B.C. : $\chi(0) = \chi''(0) = 0$ and $\chi(r \to \infty) = r\sqrt{n_0}$

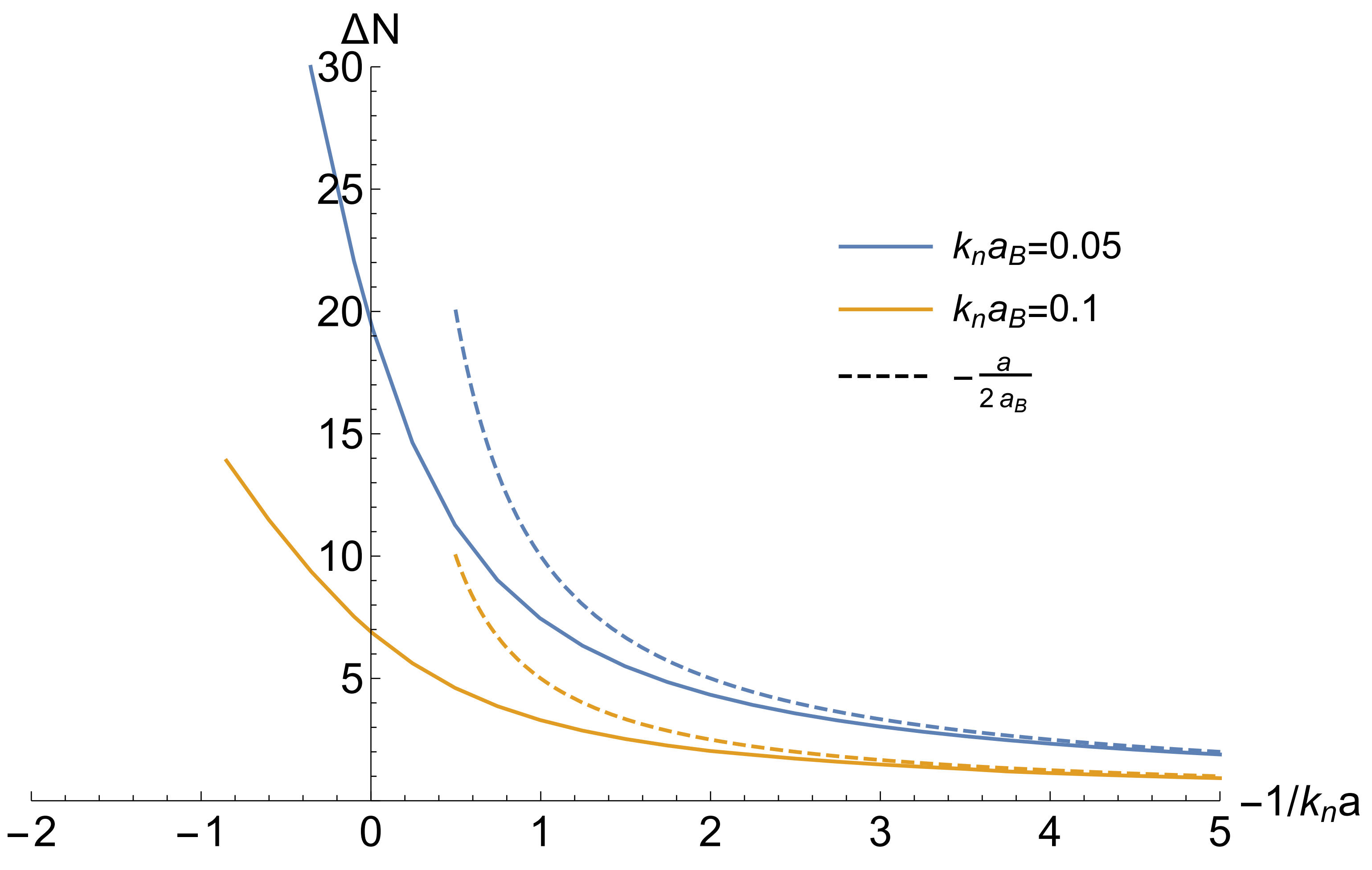
- Energy shift:



Weakly-interacting BEC

- Number of particles in the dressing cloud: $\Delta N = \int \mathrm{d}\mathbf{r}\,[n(\mathbf{r}) - n_0]\,.$
- For $|a| \lesssim a_B$ one finds $\Delta N = -\dfrac{a}{2a_B}$

Massignan, Pethick and Smith, PRA 2005



Weakly-interacting BEC

- $\psi(\mathbf{r}) = \psi_0 + \delta\psi(\mathbf{r})$
- Overlap of normalized GPE solutions: $z \equiv \left| \frac{\langle \psi_0 | \psi \rangle}{\sqrt{N_0 N}} \right|^2 = 1 - \frac{c}{N_0} + \mathcal{O}(N_0^{-2})$

  where $c \equiv \int d\mathbf{r}\ [\delta\psi(\mathbf{r})]^2 > 0$
- Many-body overlap: $Z = \lim_{N_0 \to \infty} z^{N_0} = \lim_{N_0 \to \infty} \left(1 - \frac{c}{N_0} + \mathcal{O}(N_0^{-2}) \right)^{N_0} = e^{-c}$
- Exponentially small residue ➜ Anderson orthogonality catastrophe!

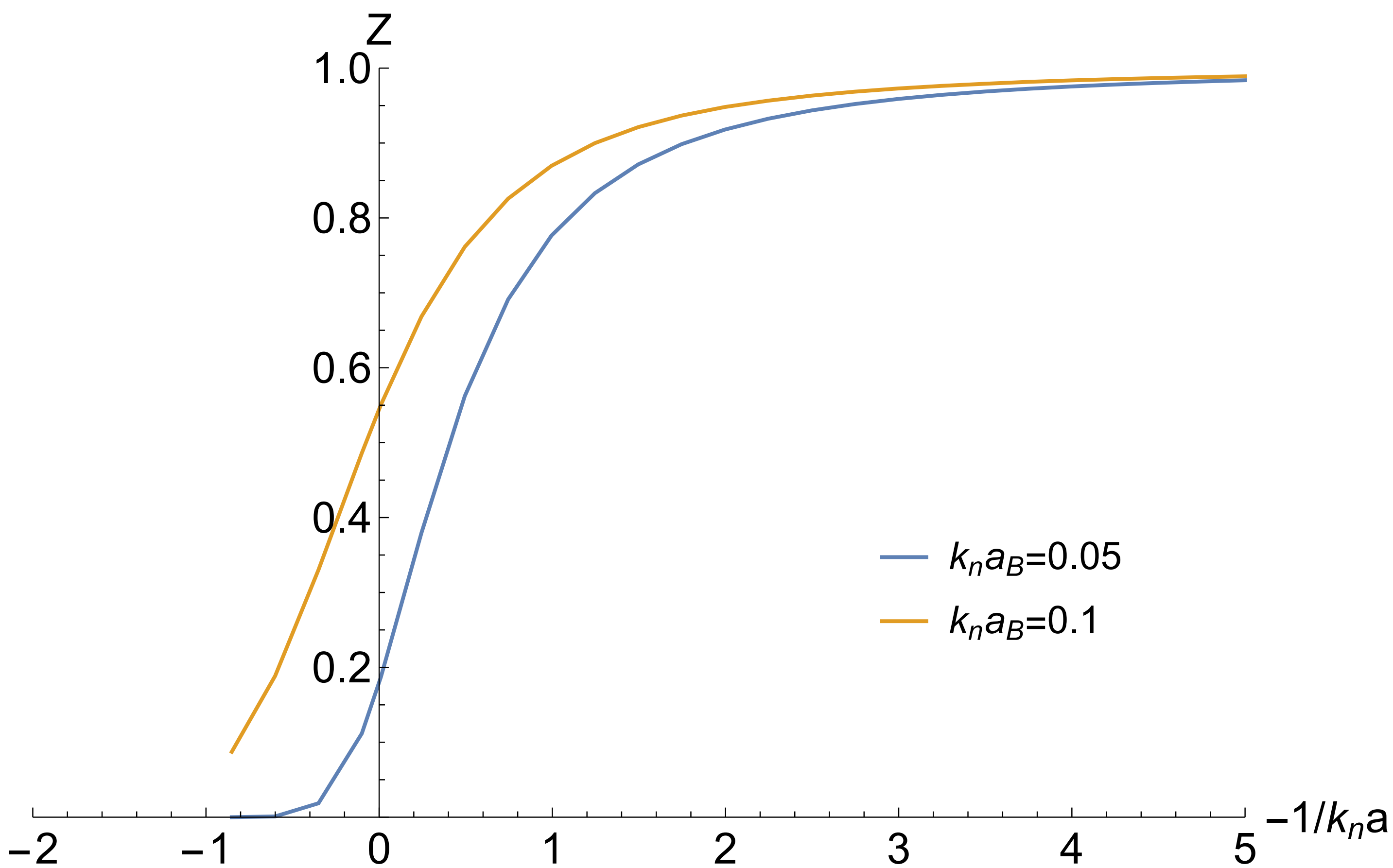
$$Z = e^{-\alpha N^{1/3}} = e^{-\beta n_0 a^2 R}$$

Weakly-interacting BEC

- Linearized GPE: $\psi_{\text{lin}}(r) = \sqrt{n_0}\left[1 + C\frac{\exp(-\sqrt{2}r/\xi)}{r}\right]$

- $Z_{\text{lin}} = e^{-c_{\text{lin}}} \approx 1 - \sqrt{2}\pi n_0 a^2 \xi$ very close to $Z_{\text{pert}} = 1 - 4\sqrt{2} n_0 a^2 \xi + \mathcal{O}(n_0 a^3)$

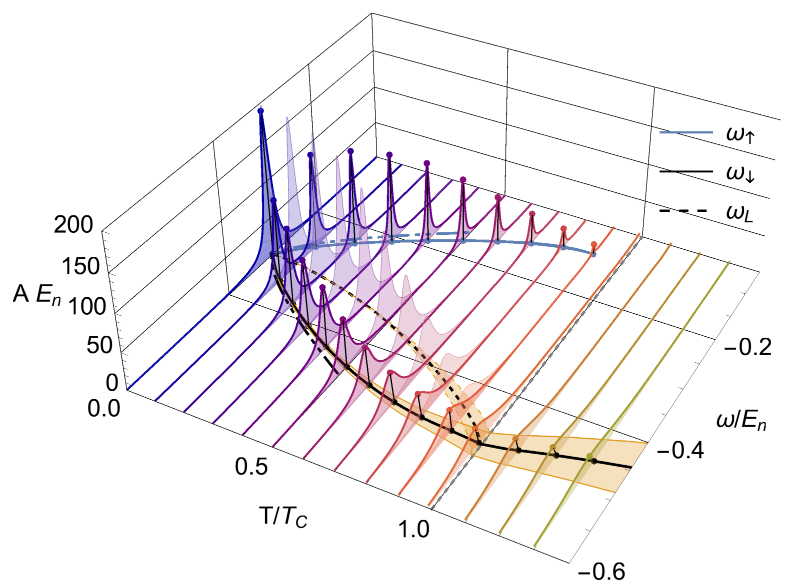
Christensen, Levinsen & Bruun, PRL 2015



Conclusions

- Bose polarons greatly differ from Fermi ones
- Crucial role played by the large low-energy density of states of the BEC
- Non-perturbative treatment is fundamental
- The T=0 attractive polaron fragments into two quasiparticles at T>0
- The ground state quasiparticle remains well-defined across T_c
- The excitation above it instead disappears at T_c

- An *Anderson orthogonality catastrophe* arises when $a_B \to 0$



Thank you!